A NOVEL APPROACH FOR JOINT ESTIMATION OF TIME DELAY AND SCALE FACTOR WITH APPLICATIONS TO THE M-WAVE ANALYSIS

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Abstract- **In this paper we propose a method for joint estimation of delay and scale factor between two deterministic and unknown signals. The method is based on the separation of the two parameters. The scale factor is estimated from the autocorrelation functions (ACFs) of the two signals. The ACFs are independent on the delay while they maintain the scale factor. The delay is estimated by scaling one of the two signals by the estimated scale factor, so that the delay estimation is not biased by the scale factor. The method proposed is compared with the maximum likelihood joint estimation and provides better performance for a rather large range of signal to noise ratios. Potential applications of the technique in surface EMG signal analysis are discussed and results related to the M-wave processing are reported.**

Keywords **- electrically elicited contractions, joint scale-delay estimators, motor unit conduction velocity distribution, M-wave, scale factor estimation**

I. INTRODUCTION

Many works are reported in literature in the area of Radar and passive Sonar to jointly estimate the scale and time delay between two or more signals. The most common approach is the maximization of the ambiguity function of the two parameters [6]. The problem usually encountered is the fact that the ambiguity function depends on signal type and model [6]. Several authors used the decorrelation method to estimate the two parameters but this approach needs a long observation window [9].

Surface EMG signals detected during electrically elicited muscle contractions represent the synchronous summation of the action potentials generated by the active muscle fibers. These signals are quasi periodic since at each electrical stimulus a compound potential (called M-wave) is generated. The scale factor and shape of each M-wave depend on the conduction velocity (CV) distribution of the active Motor Units (MUs) and on the properties of the intra-cellular action potentials. The source characteristics may change with fatigue and this is usually monitored by spectral analysis of the M-waves or estimation of CV by estimating the delay between two signals in two different locations along the muscle. Spectral variables and CV estimated during time in sustained electrically elicited contractions provide some basic indication about the decrease in velocity of propagation of the muscle fibers. Since in this case the central nervous system does not regulate the firings of the MUs, this acquisition modality is often used to investigate peripheral phenomena of muscle fatigue.

In this work we propose a novel approach to estimate jointly the time scale factor and time delay. The method is based on a first step, which aims at the estimation of the scale factor independently from the time delay, and a second step in which the scale factor between the two signals is compensated by dilatation or compression of one of the two signals and the time delay is estimated without the bias due to the scale factor. Potential applications of this method to surface EMG signal analysis are proposed and some results on M-wave analysis are reported.

II. METHODOLOGY

A. Parameter separation

An analytical model of two signals delayed and scaled is the following:

$$\begin{aligned} x_1(t) &= s(t) + w_1(t) \\ x_2(t) &= s(\alpha t - d) + w_2(t) \end{aligned} \quad (1)$$

where α and d are the scale factor and time delay, respectively, and $w_1(t)$ and $w_2(t)$ are independent, white, zero mean, additive gaussian noises with equal variance σ^2.

A solution for the problem of joint estimation of scale and delay factor is to separate the two parameters (scale and delay), to estimate one of them first and then to estimate the second after correction for the first parameter. To estimate the parameters separately, the first step consists in the estimation of the scale factor using the temporal autocorrelation functions (ACFs) of the signals. Indeed, the ACF $R_{xx}(\tau)$ of a signal preserves the scale factor and is insensitive to the time delay. For the model in (1) we get:

$$R_{x_2x_2}(\tau) = \frac{1}{\alpha} R_{ss}(\alpha\tau) + \sigma^2 \delta(\tau)$$

where δ is the Dirac distribution, σ^2 is the white noise power, and $R_{ss}(\tau)$ is the temporal autocorrelation function of $s(t)$. The effect of white noise can be reduced by neglecting the value of $R_{x_2x_2}(\tau)$ at $\tau=0$ and then interpolating this point using the neighbouring points and the best second order polynomial fit. To get ride of the multiplication factor $1/\alpha$, an amplitude normalization is then carried out. After the interpolation and normalization we get (in discrete time domain):

$$\hat{R}_{x_2x_2}(l) \cong \frac{1}{\alpha} \hat{R}_{ss}(\alpha l)$$

where ^ is for estimated values. In the same way we obtain:

$$\hat{R}_{x_1x_1}(l) \cong \hat{R}_{ss}(l)$$

To estimate the scale factor between $R_{x_1x_1}(l)$ and $R_{x_2x_2}(l)$, the value of α leading to the minimum mean square error

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Abstract

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between the two scaled ACFs can be used. A problem of this method is the limited estimation resolution. To obtain high resolution, small steps for α must be in fact chosen. Resolution can be enhanced by interpolating the mean square error function around the minimum by a second order polynomial approximation but this interpolation requires the symmetry of the function around its minimum and it can be shown that the mean square error function is not symmetric in this case [8]. To avoid the problems of calculation time and loss of resolution, other methods using the scale domain, discussed below, can be proposed.

B. Scale transform

The scale transform of a signal, introduced firstly by Cohen [1], provides the representation of a support limited signal *s(t)* in the scale domain. The scale transform is defined as:

$$D_s(c) = \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} s(t) \frac{e^{-jc\ln t}}{\sqrt{t}} dt \qquad (2)$$

After a change of variable $t \to \Delta t e^u$, the transform in Eq. (2) can be rewritten as [8]:

$$D_s(c) = \frac{\sqrt{\Delta t}}{\sqrt{2\pi}} e^{-jc\ln\Delta t} \left(\int_{-\infty}^{0} e^{u/2} s(\Delta t e^u) e^{-jcu} du + \right. \qquad (3)$$
$$\left. + \int_0^{+\infty} e^{u/2} s(\Delta t e^u) e^{-jcu} du \right)$$

where the first integral corresponds to the period of the signal *s(t)* between *s(0)* and the first sample *s(1)* of the sampled version of the signal *s(t)*. In Eq. (3) the terms between parenthesis show that the scale transform is the Fourier Transform of the exponentially resampled and weighted version of the signal *s(t)*. Moreover, if the signal *s(t)* is scaled and energy normalized (see Eq. 4 below), the scale transform is equivalent to the Fourier transform of a new function *S* that is translated by the amount $\ln\alpha$ (Eq. 5). This new translated function is obtained by the exponential resampling and weighting:

$$\sqrt{\alpha} s(\alpha t) \overset{\exp.resampling}{\Rightarrow} e^{t+\ln\alpha} s(e^{t+\ln\alpha}) = S(t+\ln\alpha) \qquad (4)$$

The Fourier transform of Eq. (4) is:

$$S(t+\ln\alpha) \overset{\textbf{F.T.}}{\Leftrightarrow} e^{jc\ln\alpha} D_s(c) \qquad (5)$$

where **F.T.** stands for the Fourier transform.

C. Estimation of scale factor from the scale transforms of the autocorrelation functions of the two signals

Eq. (5) shows the possibility of estimating the scale factor α by a spectral matching technique in the scale domain between a reference signal and its scaled version. The advantage of this method is the high resolution estimate of α and the rapid calculation time by using gradient methods for the spectral matching [7]. A drawback of this method is the presence of local minima. To avoid local minima we can choose a starting value (coarse value) of α close to the exact value. The coarse estimation can be obtained by the maximum point of the cross-correlation function between the exponentially resampled and weighted scale transforms.

After scale estimation, the scale factor can be compensated by scaling one of the two signals, for example by inverting the scale transform or by using *sinc* functions. Then the delay is estimated as the maximum point of the cross-correlation function (with a parabola fitting the apex).

Fig. 1 shows the schematic representation of the complete method used for the joint scale-delay estimation.

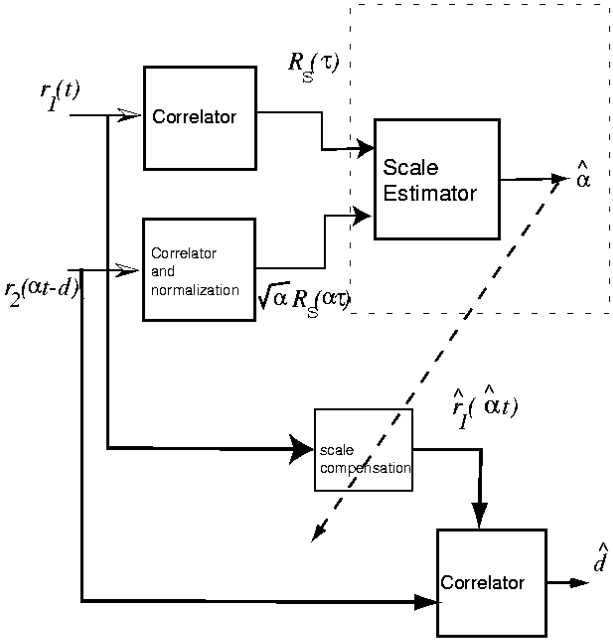


Fig. 1. Schematic representation of the method for joint time delay and scale estimation. See text for details.

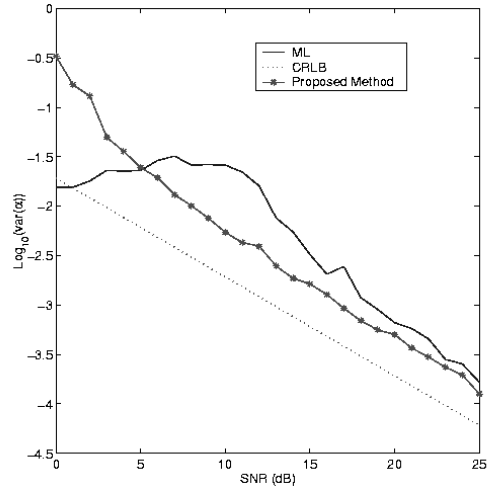
III. VALIDATION OF THE METHOD ON SIMULATED SIGNALS AND POTENTIAL APPLICATIONS IN SURFACE EMG SIGNAL ANALYSIS

A. Validation of the method on synthetic single MUAP signals

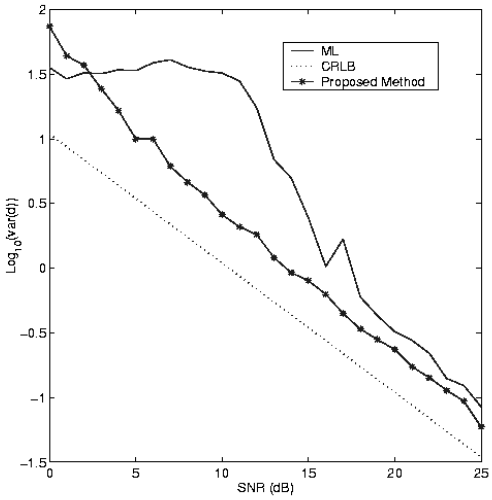
Fig. 2 shows the results of the application of the method on second order Hermite-Rodriguez functions corrupted by additive white gaussian noise (scale estimation (a), and time delay estimation (b)). The method proposed is compared with the exhaustive Maximum Likelihood (ML) and the Cramer-Rao lower bound (CRLB).

B. Potential applications for surface EMG signal analysis

The estimation of scale and delay may have useful applications in surface EMG signal analysis both during voluntary and electrically elicited contractions. The detection of single MU multi-channel EMG signals during voluntary contractions allows estimation of CV of individual MUs [2][3]. The estimation of the scale factor between MUAPs detected by different electrode systems along the muscle fibers may give an indication of fiber inclination with respect to the detection array and may be useful for electrode placement or for compensating the bias in CV estimation due to the inclination angle. The same technique can be used for precise electrode placement when a real time averaging of the surface detected EMG signals is performed with intra-muscular detected potentials as triggers [5].



(a)



(b)

Fig. 2. Variance of estimation of the scale factor and delay with the proposed method and the Maximum Likelihood joint estimation for different signal to noise ratios. The Cramér Rao lower bound is also shown. The simulated signals are Hermite-Rodriguez functions of the second order which approximate real MUAPs detected in double differential mode. The MUAP duration is approximately 12 ms and the sampling rate is 2048 Hz. The scale factor is α=0.9 and the time delay is of 6 samples.

In case of electrically elicited contractions, the estimation of scale factor between consecutive in time M-waves gives indications about the change of mean CV, as alternative to spectral analysis or CV estimation. Moreover, since the scale factor of an M-wave depends on the CV distribution, it can be used as indicative of fiber type. In particular, detecting the EMG signal along the muscle fibers during stimulation, the different M-waves will have different scale factors due to the increasing spread of the compound potential as a consequence of the different time delays of the MUAPs constituting the M-wave. Thus estimation of the time delay between compound potentials during time gives indication about mean CV and estimation of scale factor between M-waves detected at the same instant of time in two different locations in space gives indication of CV distribution spread and its changing during time. Moreover, the estimation of delay between consecutive in time M-waves gives also an indication of mean CV if the estimation is not biased by the scale factor.

C. Simulation of electrically elicited surface EMG signals

EMG signals detected during electrically elicited contractions were simulated with the model proposed by Farina & Merletti [4]. MUs were placed randomly in the detection volume. Simulated signals were detected with three single differential recording systems placed along the fiber direction. Two double differential signals were computed as the difference between consecutive single differential recordings. Different values of CV distribution mean and standard deviation were simulated. The scale factor between M-waves detected in double differential mode from consecutive recording systems was estimated. As expected, it has been found that the scale factor estimation is insensitive from the thickness of subcutaneous tissue layers. On the other hand, the scale factor indicates the different CV distribution standard deviations. Statistically different values of scale factors between M-waves have been found for differences in CV distribution standard deviation lower than 0.03 m/s. Fig. 3 reports the results obtained in one simulation set.

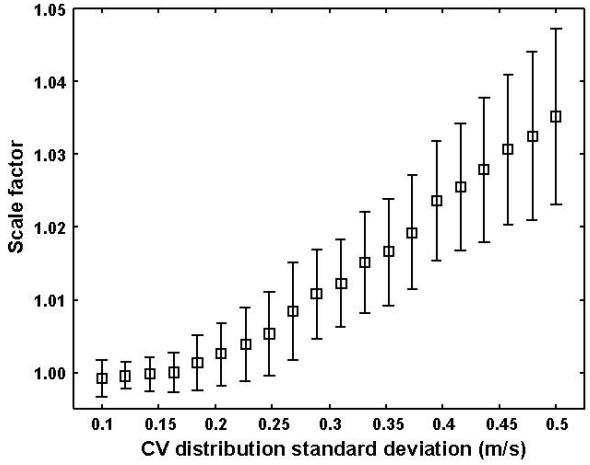


Fig. 3. Scale factor (mean ± standard deviation) between simulated M-waves detected in single differential mode at a distance of 1 cm along the muscle fibers. Fifty simulations have been performed for each CV distribution standard deviation value. In each simulation the location of the MUs in the muscle was randomly changed in the detection volume in order to assess the variability of the estimates due to the volume conductor masking effect.

V. DISCUSSION

The estimation of delay and scale factor between two deterministic signals is not trivial since the two parameters are correlated and a joint estimation is necessary. We have proposed a method for the joint estimation which allows to reach good performance with limited computational time. The performances of the method are better than those of an exhaustive ML based estimation even if we take a high resolution search grid on the ambiguity function.

The technique can be applied in many fields. Potential applications to surface EMG signal analysis have been discussed and results on simulated electrically elicited contractions have been reported. In particular, the estimation

of the scale factor between M-waves detected along the muscle fibers may give indication about the distribution of CVs with which the action potentials propagate. Since the measure is not absolute but relative (scale difference between two M-waves), it is insensitive to the thickness of subcutaneous tissue layers, to mean CV or to the length of the intra-cellular action potentials; these factors, in fact, affect in the same way the width of both M-waves. On the other hand this estimate may be affected by fiber inclination with respect to the detection system.

Although the possibility of quantitatively associate to a scale factor estimate a corresponding CV distribution standard deviation value should still be investigated, the presented results shows the feasibility of using the scale factor as a variable for relative comparisons between subjects and muscles. A validation of the technique on experimental signals will be the subject of further investigation.

VI. Conclusions

A method for joint scale and delay estimation and its potential applications to the analysis of electrically elicited EMG signals has been proposed. Performances have been statistically analysed (Monte Carlo method) giving better result in comparison to ML approach. In order to investigate the feasibility of applying the proposed method to real cases, simulated signals have been generated leading to promising results.

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